

## Function Verification Using the Derivative

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**Annotation:** there is a relationship between the character of change of a function and its derivative, and a number of properties belonging to the nature of function can be determined using the derivative.

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There is a link between the change character of a function and its derivative, and a number of properties belonging to the nature of the function can be determined using the derivative.

Given a function  $U = f(x)$  in the range  $V = [a; b]$ , for two numbers  $x_1$  and  $x_2$  that can be chosen from any given range, we recall that for  $x_1 < x_2$  the relation  $F(x_1) < f(x_2)$  ( $f(x_1) > f(x_2)$ ) inequality follows, then  $u = f(x)$  the function  $V$  is said to be increasing decreasing.

Let  $V = [a; b]$  be a function  $U = f(x)$  defined in the cut, continuous in that cut, and differentiable in the interval  $(a; b)$ . A sufficient condition for the function to increase (or decrease) in the range  $V$  consists of the following theorem.

**Theorem 1.** In order for a differentiable function  $f(x)$  in the range  $V$  to be increasing (decreasing) in that range, it is sufficient for the derivative  $P(x)$  at each inner point of the range to be positive (negative).

Let no points  $x_1$  and  $x_2$  belonging to the range  $X$  be considered, for a function  $f(x)$  in the  $[x_1; x_2]$  cut, Lagrange's theorem is appropriate, i.e.,  $f(x_2) - f(x_1) = f'(c) (x_2 - x_1)$ , where  $x_1 < x_2$  and  $s \in (x_1; x_2)$ . From equality, it follows that if  $f'(c) > 0$ ,  $f(x_2) > f(x_1)$  and the function is increasing, and if  $f'(c) < 0$ ,  $f(x_2) < f(x_1)$  and the function is decreasing.

A geometric explanation of the symptoms of monotony of the function is given.

a)  $f'(c_1) = \tan \alpha > 0$

b)  $f'(c_2) = \tan \alpha < 0$

$y = f(x)$  attempts transferred to a function graph are reducible if  $X$  forms an acute angle with positive direction of the  $OX$  axis at Intermediate Interior points, while the function produces a growing, non-transitive angle.

Issue.  $y = x \cdot e^{-2x}$  check the function for monotony.

A given function is defined in  $R$  and at each point  $x \in R$   $y'(x) = e^{-2x} \cdot (1 - 2x)$  has a derivative and is differentiable. If  $x < 1/2$  while,  $y'(x) > 0$  is a function-increasing, if  $x > 1/2$  bo'lsa,  $y'(x) < 0$  is a function reducible. So,  $y = x \cdot e^{-2x}$  function  $(-\infty; 1/2)$  monotone growing in the range,  $(1/2; \infty)$

the range, however, is a monotone reducer.

**Issue.**  $f(x) = x - \arctg x$  prove that the function is a growth on the number axis.

$f'(x) = (x - \arctg x)' = 1 - 1/(1+x^2)$ , for each  $x \in \mathbb{R}$ ,  $f'(x) > 0$ . Hence, the function is a monotone increment at point R.

## 2. Function extremes. Necessary and sufficient conditions of extremum:

$y = f(x)$  the function is defined around some  $\delta$  of point  $x_0$ , so that it is continuous at point  $x_0$ .

If all  $x \in (x_0 - \delta; x_0) \cup (x_0; x_0 + \delta)$  for points  $f(x) < f(x_0)$  ( $f(x) > f(x_0)$ ) where the inequality is appropriate,  $x_0$  is called the fixed maximum (minimum) point of the function.

If, for every  $x \in (x_0 - \delta; x_0) \cup (x_0; x_0 + \delta)$ , the inequality  $f(x) < f(x_0)$  ( $f(x) > f(x_0)$ ) is satisfied, then  $x_0$  is called the non-linear maximum (minimum) point of the function.

The fixed and non-linear maximum and minimum points of a function are called the extremum points of its local (local) nature.

If  $x_0$  is the maximum point of a function  $f(x)$ , then the relationship  $\Delta f(x_0) = f(x) - f(x_0) < 0$  ( $\Delta f(x_0) < 0$ ) is appropriate around the Looking 6 of point  $x_0$ . If  $x_0$  is the minimum point of the function  $f(x)$ , then the inequalities  $\Delta f(x_0) > 0$  ( $\Delta f(x_0) > 0$ ) are fulfilled.

Satisfying the necessary conditions of the function extremum, i.e. that the function derivative  $F(x)$  becomes zero, or  $F'(x)$  exists, the inner points of the function detection domain are called its critical points. Of these, the critical points satisfying the equation  $f'(x) = 0$  are called stationary points.

**Example.**  $y = (x-4) \cdot x^3$  find the critical points of the function.

The function is defined on the number axis and  $y'(x) = 4/3 \cdot x - 1/3 \cdot x^3$ . at  $x = 1$ ,  $y'(1) = 0$ , and at  $x = 0$ ,  $y'(0)$  - does not exist.

Hence, point  $x = 1$  is the stationary point of the function, and point set  $\{0;1\}$  is the set of its critical points.

Not every critical point satisfying the necessary condition of a function's extremum is its extremum point. For example, a function  $u = x^3$  is monotone-increasing in  $\mathbb{R}$  because  $(x^3)' \geq 0$ ,  $x \in \mathbb{R}$ . and point  $x = 0$  is its critical (stationary) point because  $y'(0) = 0$ . Since the function is a monotone growth on the number axis, the critical point  $x = 0$  cannot be its extremum.

The extremum points of a function are chosen from within its critical points based on one of the following sufficient conditions.

Above was defined the set of critical points of the function  $\{0;1\}$ . In the field of function determination, we divide the number axis into intervals using critical points and check the sufficient conditions:

Hence, critical point  $x = 0$  is not an extremum point, and point  $x = 1$  is the minimum point of the function,  $y(1) = -3$ .

**Theorem 4.** (2-sufficient condition) is  $f(x_0) = 0$  if there exists a second order derivative  $f''(x_0)$  at the stationary point  $x_0$ , then if  $f'(x_0) < 0$ . the  $x_0$  - maximum point, if  $f''(x_0) > 0$ , is the  $x_0$ -minimum point, and if  $f''(x_0) = 0$ , the existence question of the extremum at  $x_0$  remains open.

**Issue.** find the extremum points of the function  $u = x^3 + 6x^2$ .

The equation solutions  $y' = 3 - (x^2 + 4x)$  and  $y'(x) = 0$  for the function derivative  $X = -4$ ,  $x = 0$  are its stationary points. The second order derivative is  $y'' = 6 - (x + 2)$ . Y in stationary units  $y''(-4) = -12 < 0$ ,  $y''(0) = 12 > 0$  for, according to the latter condition  $x = -4$  is the fixed maximum point and  $y(-4) = 32$ ,  $x = 0$  is the fixed minimum point and  $y(0) = 0$ .

**Theorem 5.** (3 is a sufficient condition) for a function  $f(x)$ , point  $x_0$  and in turn let  $f'(x_0) = f''(x_0) = \dots = f^{(n-1)}(x_0) = 0$  be the Equalities appropriate and let  $f^{(n)}(x_0) \neq 0$ . In it:

a) if  $N$  is even and  $f^{(n)}(x_0) < 0$ ,  $x_0$  is a fixed maximum point,  $f^{(n)}(x_0) > 0$ ,  $x_0$  is a fixed minimum point;

(b) if  $n$  is odd, there will be no  $x_0$  - extremum point.

For example, for a function  $U = X^4$ ,  $y'(x) = 4x^3$ ,  $y''(x) = 12x^2$ ,  $y'''(x) = 24x$ ,  $y''''(x) = 24$ . since the equation solution  $y' = 0$  is  $y'(0) = y''(0) = y'''(0) = 0$  and  $y''''(0) = 24 > 0$  at the stationary point  $x = 0$ , according to the third sufficient condition  $x = 0$  is the fixed minimum point and  $y(0) = 0.3$ .

### **The largest and smallest values of the function in the set are:**

In applied economics, particularly optimization issues, it is important to find the largest and smallest values of a function in set  $V$ , i.e. global extremes.

The one - variable function  $y = f(x)$  is defined on some  $V \in \mathbb{R}$ , set, and let  $x_0 \in V$ .

If the inequality  $f(x) \leq f(x_0)$  is performed for all  $x \in V$ , at point  $x_0$ , the function  $f(x)$  takes its largest  $f_{\max} = f(x_0)$  value, and vice versa, if for each  $x \in V$  the relation  $f(x) > f(x_0)$  is appropriate, then at Point  $x$ , the function  $f(x)$  is said to achieve its smallest  $f_{\min} = f(x_0)$  value.

If the function  $y = f(x)$  is continuous on the cross section  $V = [a;b]$ , according to one of the properties of the continuous function on the compact set it assumes its largest and smallest values in this section. The function can achieve its global extremes not only at the extremum points belonging to the section, but also at its edge points.

To find the largest and smallest values of the function in the section:

- a) the critical points of the function belonging to the section are defined;
- b) are the values of the function at the found critical points and at the edge points of the section;
- c) these values are compared among themselves and the largest, smallest are selected.

**Issue.**  $f(x) = x + 1/x$ ,  $[0.01;10]$  largest and smallest in cross section find the values.

$f'(x) = (x + 1/x)' = 1 - 1/x^2$ ,  $x = \pm 1$  points are stationary points of the function. Of these, Point  $x = 1$  is the only stationary point belonging to the section.

Thus, we calculate the function values at points  $x = 0.01$ ,  $x = 1$  and  $x = 10$ :

$f(0.01) = 100.01$ ;  $f(1) = 2$ ;  $f(10) = 10.1$ . Hence, in the section under consideration, the global minimum of the function is at Point  $x = 1$ , with  $f_{\min} = f(1) = 2$ , and  $x = 0.01$  being its global maximum and  $f_{\max} = f(0.01) = 100.01$ .

If a function has breakpoints in the section under consideration, further, function breakpoints are added to the above. Given a function  $(A;b)$  in an interval, it is required to check the function from the right at point  $A$ , and from the left at Point  $B$ .

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