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Solving a One-Dimensional Oil-Gas Two-Phase Flow Problem in Porous Media Using Numerical Methods

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Abstract

Modeling of oil and gas reservoirs on a computational platform requires a precise mathematical model of the system, which describes the fluid flow in porous media based on fundamental physical laws governing their movement within the reservoir. Currently, well-established mathematical models exist that describe the multiphase filtration processes of fluids in porous media. Using the filtration theory equations, this work derives a system of equations that describes the transient filtration processes of oil and gas in a reservoir. The model provides a theoretical basis for simulating complex multiphase flows, which are critical for optimizing reservoir management and production strategies.

Keywords: Oil-gas reservoir, porous medium, multiphase filtration, mathematical modeling, fluid flow, transient filtration, filtration theory, numerical simulation.

Introduction

The efficient development and management of oil and gas reservoirs heavily depend on a thorough understanding of fluid flow behavior within porous geological formations. Due to the complex nature of multiphase fluid interactions in porous media, accurate mathematical modeling becomes essential for predicting reservoir performance and optimizing extraction processes.

Computational simulation of these processes requires robust mathematical frameworks that reflect the physical laws governing fluid dynamics, phase interactions, and filtration mechanisms in reservoir rocks. Over the past decades, numerous mathematical models have been proposed to describe multiphase filtration in porous media, contributing significantly to enhanced reservoir characterization and production forecasting. This study focuses on deriving and analyzing a system of governing equations based on filtration theory to simulate the transient multiphase flow of oil and gas in porous reservoirs. The presented model aims to improve the predictive capabilities for reservoir behavior under varying operational conditions, thereby contributing to more efficient resource management.

Simulation of oil or gas formations on computational devices necessitates a comprehensive mathematical model of the system, which accurately reflects the behavior of fluids in porous media as determined by fundamental laws governing their flow within the formation. To date, mathematical models describing multiphase fluid filtration in porous environments are well documented. Pioneering contributions to these models have been provided by experts including N.N. Veregin, V.N. Nikolaevskiy, V.M. Shestakov, E.S. Zakirov, B.B. Lapuk, F.B. Abutaliyev, D.F. Fayzullaev, R. Sadullaev, among others. Utilizing the governing equations of filtration theory, we obtain a set of equations that depict the non-stationary filtration phenomena of oil and gas in reservoirs.

$$\begin{cases} \frac{\partial}{\partial x}\left[\lambda_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{\partial z}{\partial x}\right)\right]=\frac{\partial}{\partial t}\left[m\rho_o\left(1-S_g\right)\right], \\ \frac{\partial}{\partial x}\left[R_s\lambda_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{\partial z}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left[\lambda_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{\partial z}{\partial x}\right)\right] \\ =\frac{\partial}{\partial t}\left[m\rho_o R_s\left(1-S_g\right)+m\rho_g S_g\right]+(R_s q_o+q_g)\delta_i, \\ P_g-P_o=P_{cog},\ \ S_g+S_o=1. \end{cases} \quad (1)$$

Here,

(l= 0, g)— conductivity of the *l*-phase;

K, r,— relative permeability for the *l* reservoir -phase;

k— absolute permeability;

m— porosity of the;

μ_l — viscosity of the l-phase;

ρ_l— density of the l-phase;

Rs— oil solubility in gas;

z— distance from a certain plane;

q, *l* — volumetric flow rate withdrawn by the well for the *l*-phase;

γ, *l* — specific weight of the *l*-phase.

For the sake of convenience in notation, we assume that...

$$\sum_{i=1}^{n} q_{o_i}\delta(x-\xi_i)=q_o,\ \ \sum_{i=1}^{n} q_{g_i}\delta(x-\xi_i)=q_g.$$

To close the system of equations, the following initial conditions are specified:

$$\begin{cases} P_o(x,0)=P_o^H(x), \quad P_g(x,0)=P_g^H(x), \\ S_o(x,0)=S_o^H(x), \quad S_g(x,0)=S_g^H(x) \end{cases} \quad (2)$$

and boundary conditions of the form

$$\frac{\partial P_l}{\partial x}\Big|_{x=0}=0, \qquad \frac{\partial P_l}{\partial x}\Big|_{x=L}=0 \quad (3)$$

or

$$P_l\Big|_{x=0}=P_l^H(t), \qquad P_l\Big|_{x=L}=P_l^H(t),$$

(4)

Where

L - is the boundary of the filtration domain.

In the filtration mathematical model of the oil-gas system, the capillary pressure between oil and gas is determined empirically as a function of gas saturation. Let us assume that oil is incompressible, i.e., ρoil= constant, while gas is compressible, with its density expressed as a function of pressure through the equation of state, i.e.,

$$\rho_g = \frac{P_g}{RTZ} \qquad (5)$$

where

R - the universal gas constant,

T - the temperature

Z - the gas compressibility factor.

The problem (1)– (5) was solved using a combined finite difference and iterative method. Computational experiments were conducted for various values of reservoir permeability coefficients, oil and gas viscosities, as well as well flow rates under symmetrical configuration. The redistribution of pressure and saturation fields of oil and gas in the reservoir over time was investigated.For determining the relative phase permeabilities, the following dependencies based on experimental results obtained by *UZBEKNEFTEGAZ* for the oil-gas system were used.

$$K_o = 0.839379\ S_g^3 + 1.12471\ S_g^2 - 1.0396\,S_g + 0.182166,$$

$$K_g = -3.27135\ S_g^3 + 7.73761\ S_g^2 - 6.25468\ S_g + 1.73322,$$

The gas solubility in oil is expressed as: $R_s = 11.3 + 0.75\,P_o$;

The reservoir is assumed to be horizontal, and the effect of gravitational forces is considered negligible.

In all calculations, the following values for reservoir parameters and boundary conditions were used:

$L_x = 10^4 m$; $m = 0.1$; $H = 20m$; $K^H = 0.1$ дарси; $P^H = 300$ атм.; $P_o = 0.87$ г/см^3; $P_g = 0.82$ г/см^3; $P_w =$ г/см^3;

R = 8.31 Дж/(моль К); T=273 К; $P_l^0 = 300$; $S_g^0 = 0.8$; $S_o^0 = 0.2$.

Based on the conducted computational experiments, the influence of parameter variations on the distribution of phase pressures and... was identified.

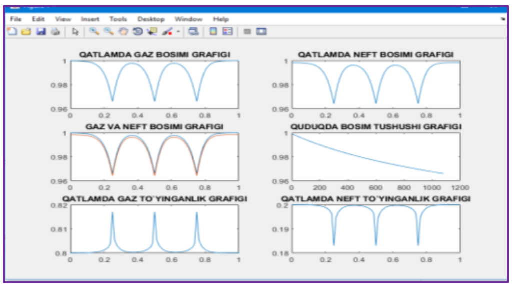

Fig. 1. Distribution of gas and oil pressure in the reservoir and the corresponding saturations at $K = 0.1$, $\mu_o = 4cП$, $\mu_g = 0.01cП$.

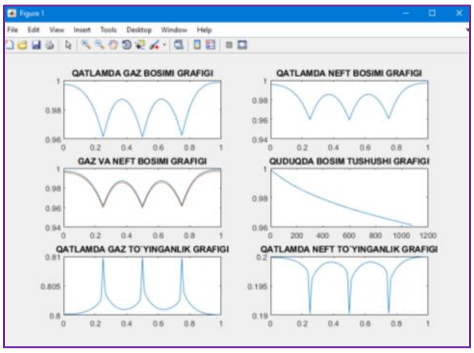

Fig. 2. Distribution of gas and oil pressure in the reservoir and the corresponding saturations at $K = 0.2$, $\mu_o = 4cП$, $\mu_g = 0.01cП$.

Saturations, as well as pressure drops at the wells.

Analysis of the results showed that these parameters have a significant impact on the distribution of oil and gas pressures within the reservoir and on the phase saturations.

Conclusion

The conducted numerical simulations confirm the effectiveness of the proposed mathematical model and computational algorithm for describing the unsteady, multiphase filtration processes occurring during the simultaneous flow of oil and gas in porous media.

The results demonstrate that variations in key reservoir parameters such as permeability, phase viscosities, and well production rates significantly affect the distribution of pressures and saturations within the reservoir. Furthermore, the model allows for the analysis of pressure drops at production wells and dynamic changes in fluid distribution over time.

These findings validate the applicability of the developed approach for evaluating filtration behavior in oil-gas systems and can be effectively utilized in the design, simulation, and optimization of oil and gas field development projects.

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